DAY —— 06 SEAT NUMBER

2017	III	06	1100	J - 541	(E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs. **(7 Pages)** **Max. Marks : 80**

Note : *(i)* *All questions are compulsory.*

(ii) *Figures to the right indicate full marks.*

(iii) *Graph of L.P.P. should be drawn on graph paper only.*

(iv) *Use of logarithmic table is allowed.*

(v) *Answers to the questions of both sections should be written in the same answer book.*

(vi) *Answer to every new question must be written on a new page.*

SECTION – I

Q. 1. **(A)** Select and write the appropriate answer from the given alternatives in each of the following sub-questions : **(6)** **[12]**

(i) If the points A(2, 1, 1), B(0, –1, 4) and C(k, 3, –2) are collinear, then k = ______.

(a) 0

(b) 1

(c) 4

(d) – 4 (2)

(ii) The inverse of the matrix $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$ is ______.

(a) $\frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$ (b) $\frac{1}{13}\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

(c) $\frac{1}{13}\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$ (d) $\frac{1}{13}\begin{bmatrix} 1 & 5 \\ 3 & -2 \end{bmatrix}$ (2)

(iii) In Δ ABC, if $a = 13$, $b = 14$ and $c = 15$, then $\sin\left(\frac{A}{2}\right) =$ ______.

(a) $\frac{1}{5}$ (b) $\sqrt{\frac{1}{5}}$

(c) $\frac{4}{5}$ (d) $\frac{2}{5}$ (2)

(B) Attempt any THREE of the following : **(6)**

(i) Find the volume of the parallelopiped whose coterminus edges are given by vectors $2\hat{i}+3\hat{j}-4\hat{k}$, $5\hat{i}+7\hat{j}+5\hat{k}$ and $4\hat{i}+5\hat{j}-2\hat{k}$. (2)

(ii) In ΔABC, prove that, $a\,(b \cos C - c \cos B) = b^2 - c^2$. (2)

(iii) If from a point Q (a, b, c) perpendiculars QA and QB are drawn to the YZ and ZX planes respectively, then find the vector equation of the plane OAB. (2)

(iv) Find the cartesian equation of the line passing through the points A(3, 4, –7) and B(6, –1, 1). (2)

(v) Write the following statement in symbolic form and find its truth value :

$\forall\, n \in N$, $n^2 + n$ is an even number and $n^2 - n$ is an odd number. (2)

Q. 2. (A) Attempt any TWO of the following : **(6) [14]**

(i) Using truth tables, examine whether the statement pattern $(p \wedge q) \vee (p \wedge r)$ is a tautology, contradiction or contingency. (3)

(ii) Find the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}.$$ (3)

(iii) Find the general solution of the equation

$\sin 2x + \sin 4x + \sin 6x = 0$ (3)

(B) Attempt any TWO of the following : **(8)**

(i) Solve the following equations by method of reduction :

$x - y + z = 4,\ 2x + y - 3z = 0,\ x + y + z = 2$ (4)

(ii) If θ is the measure of the acute angle between the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$ where $a + b \neq 0$ and $b \neq 0$.

Find the condition for coincident lines. (4)

(iii) Using vector method, find the incentre of the triangle whose vertices are P(0, 4, 0), Q(0, 0, 3) and R(0, 4, 3). (4)

Q. 3. (A) Attempt any TWO of the following : **(6) [14]**

(i) Construct the switching circuit for the statement $(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$. (3)

(ii) Find the joint equation of the pair of lines passing through the origin which are perpendicular respectively to the lines represented by $5x^2 + 2xy - 3y^2 = 0$. (3)

(iii) Show that $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$ (3)

(B) Attempt any TWO of the following : **(8)**

(i) If l, m, n are the direction cosines of a line, then prove that $l^2 + m^2 + n^2 = 1$. Hence find the direction angle of the line with the X axis which makes direction angles of 135° and 45° with Y and Z axes respectively. (4)

(ii) Find the vector and cartesian equations of the plane passing through the points A (1, 1, –2), B(1, 2, 1) and C(2, –1, 1). (4)

(iii) Solve the following L. P. P. by graphical method :

Maximise : $Z = 6x + 4y$

subject to $x \leq 2,\ x + y \leq 3,\ -2x + y \leq 1,\ x \geq 0,\ y \geq 0$ (4)

SECTION - II

Q. 4. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions : **[12]** **(6)**

(i) Derivative of $\tan^3\theta$ with respect to $\sec^3\theta$ at $\theta = \frac{\pi}{3}$ is ______.

(a) $\frac{3}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) $-\frac{\sqrt{3}}{2}$ (2)

(ii) The equation of tangent to the curve $y = 3x^2 - x + 1$ at P(1, 3) is ______.

(a) $5x - y = 2$ (b) $x + 5y = 16$

(c) $5x - y + 2 = 0$ (d) $5x = y$ (2)

(iii) The expected value of the number of heads obtained when three fair coins are tossed simultaneously is ______ .

(a) 1 (b) 1·5

(c) 0 (d) −1 (2)

(B) Attempt any THREE of the following : **(6)**

(i) Find $\frac{dy}{dx}$ if $x \sin y + y \sin x = 0$. (2)

(ii) Test whether the function, $f(x) = x - \frac{1}{x}, x \in R, x \neq 0$, is increasing or decreasing. (2)

(iii) Evaluate : $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$ (2)

(iv) Form the differential equation by eliminating arbitrary constants from the relation $y = A e^{5x} + B e^{-5x}$ (2)

(v) The probability that a bomb will hit a target is 0·8. Find the probability that out of 10 bombs dropped, exactly 4 will hit the target. (2)

Q. 5. (A) Attempt any TWO of the following : **(6) [14]**

(i) Solve : $\frac{dy}{dx} = \cos(x + y)$ (3)

(ii) If u and v are two functions of x, then prove that :

$$\int u\, v\, dx = u \int v\, dx - \int \left[\frac{du}{dx} \int v\, dx \right] dx$$ (3)

(iii) If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$, for $x \neq 0$, is continuous at $x = 0$,

find $f(0)$. (3)

(B) Attempt any TWO of the following : **(8)**

(i) If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and $\dfrac{dx}{dy} = \dfrac{1}{\frac{dy}{dx}}$ where $\dfrac{dy}{dx} \neq 0$.

Hence find $\dfrac{d}{dx}\left(\tan^{-1} x\right)$. (4)

(ii) A telephone company in a town has 5000 subscribers on its list and collects fixed rent charges of ₹ 3,000 per year from each subscriber. The company proposes to increase annual rent and it is believed that for every increase of one rupee in the rent, one subscriber will be discontinued. Find what increased annual rent will bring the maximum annual income to the company. (4)

(iii) Evaluate : $\displaystyle\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$ (4)

Q. 6. (A) Attempt any TWO of the following : **(6) [14]**

(i) Discuss the continuity of the following function, at $x = 0$.

$$f(x) = \frac{x}{|x|}, \text{ for } x \neq 0$$
$$= 1, \text{ for } x = 0$$

(3)

(ii) If the population of a country doubles in 60 years, in how many years will it be triple under the assumption that the rate of increase is proportional to the number of inhabitants? [Given : $\log 2 = 0{\cdot}6912$ and $\log 3 = 1{\cdot}0986$.] (3)

(iii) A fair coin is tossed 8 times. Find the probability that it shows heads

(a) exactly 5 times

(b) at least once. (3)

(B) Attempt any TWO of the following : **(8)**

(i) Evaluate : $\displaystyle\int \frac{d\theta}{\sin\theta + \sin 2\theta}$ (4)

(ii) Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (4)

(iii) Given the probability density function (p.d.f.) of a continuous random variable X as,

$$f(x) = \frac{x^2}{3}, \quad -1 < x < 2$$
$$= 0, \text{ otherwise}$$

Determine the cumulative distribution function (c.d.f.) of X and hence find P (X < 1), P (X > 0), P (1 < X < 2). (4)

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